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SEASONAL VARIATION IN THE CONCENTRATION OF COPPER IN THE SURFACE WATERS OF SAN JUAN CHANNEL, WASHINGTON[1]

By

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ABSTRACT

The concentration of copper in the surface water of San Juan Channel in the State of Washington was determined over a period of 17 months. The average concentration in all samples analyzed was 0.023 μg-at/L. A definite seasonal trend was indicated with an autumn minimum of 0.016 μg-at/L and a summer maximum of 0.028 μg-at/L. The copper content varied inversely with the values obtained for density, phosphates and silicates. On several occasions during early summer, abnormally high values of 0.060 μg-at/L were obtained at times of lower low tide, low salinity, high water temperature and when there were no meteorological disturbances. Presumably these abnormal values result from reactions that take place in areas where anaerobic conditions cause precipitation of copper sulfide, which is then oxidized to soluble copper sulfate under favorable climatic conditions. By the process of leaching and drainage such waters, because of their low densities, become surface water in the adjacent areas.

INTRODUCTION

In a recent paper, Chow and Thompson (1952) outlined a method for the determination of copper in sea water, summarized results of

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previous investigations, and gave data on a number of analyses. The present investigation was undertaken to study the seasonal variation in the concentration of copper in the surface waters of San Juan Channel near the Friday Harbor Laboratories, University of Washington, to ascertain possible correlation between the concentration of copper and other minor constituents of sea water, to observe the extent of changes in the concentration of copper during a tidal cycle, and to note variations at different depths.

METHODS OF ANALYSIS

The standardized oceanographic procedures employed in this study are outlined as follows:

Temperature: Obtained with Richter type reversing thermometers, which were graduated to 0.05° C. Readings were made to 0.01° C.

Chlorinity: Determined by the Knudsen method, the silver nitrate solution being standardized against the standard sea water of the Copenhagen Hydrographic Laboratories.

Dissolved Oxygen: Determined by the modified Winkler method (Thompson and Robinson, 1939).

Inorganic Phosphates: Determined by the refined Deniges molybdenum blue method (Robinson and Thompson, 1948a). In order to compensate for the salt effect, phosphate-free synthetic sea water was used in the preparation of comparison standards. Analyses were made as soon as samples had reached room temperature.

Silicates: Determined by the silico-molybdate colorimetric method, using potassium chromate solution buffered with borax as permanent standards (Robinson and Thompson, 1948b).

Copper: Ascertained by the diethyldithiocarbamate method (Chow and Thompson, 1952). To prevent contamination from sampling devices, all surface samples were collected in Pyrex glass bottles and the samples were immediately transferred to polyethylene containers. Only stainless steel wire was utilized when depth samples were secured with the ZoBell bacteria sampling bottle (ZoBell, 1941). Each value given for copper was the average of three determinations.

RESULTS

The monthly average oceanographic data for the surface waters of San Juan Channel are given in Table I and Fig. 1. The mean temperature for the year was 9.04° C, with a range from 6.57° to 13.10° C. The range for monthly average temperatures varied from 6.79° C in January 1952 to 11.23° and 10.63° in August 1951 and 1952 respectively.

TABLE I. Monthly Average Concentration of Copper and Various Oceanographic Data for the Surface Waters of San Juan Channel Near Friday Harbor, Washington, during 1951 to 1953.

Date	*Temp.* (°C)	*Chlorinity* (‰)	σ_t	*Dissolved Oxygen* (mg-at/L)	*Dissolved Oxygen* (% Sat'n)	*Phosphate-P* (μg-at/L)	*Silicate-Si* (μg-at/L)	*Copper* (μg-at/L)
1951								
August	11.23	16.52	22.75	0.413	72.4	1.70	27	0.028 (7)*
September	10.61	16.92	23.42	0.383	66.6	1.90	33	0.022 (29)
October	9.74	17.11	23.83	0.364	63.8	2.15	37	0.019 (8)
November	8.76	17.01	23.84	0.427	71.0	2.20	40	0.016 (7)
December	8.00	16.92	23.81	0.498	83.7	2.10	40	0.018 (7)
1952								
January	6.79	16.97	24.06	0.537	88.0	2.10	40	0.017 (6)
February	6.90	16.81	23.82	0.553	90.6	2.10	40	0.018 (6)
March	7.12	16.89	23.90	0.543	89.4	1.90	38	0.021 (7)
April	7.71	16.93	23.88	0.530	88.6	1.90	35	0.021 (7)
May	8.71	16.78	23.53	0.494	84.4	1.90	34	0.022 (7)
June	9.51	16.48	22.96	0.444	75.6	1.75	34	0.025 (8)
July	10.43	16.27	22.54	0.461	79.5	1.65	34	0.028 (11)
August	10.63	16.65	23.05	0.382	66.1	1.70	35	0.028 (8)
1953								
June	9.89	16.62	23.09	0.456	77.5	1.70	38	0.028 (7)
July	10.81	16.22	22.56	0.440	76.0	1.70	39	0.032 (27)
August	10.83	16.51	22.80	0.387	66.0	1.90	41	0.025 (22)
September	10.87	16.54	22.84	0.375	65.3	1.90	44	0.020 (9)

*Figure in parentheses after value for copper designates number of different samples.

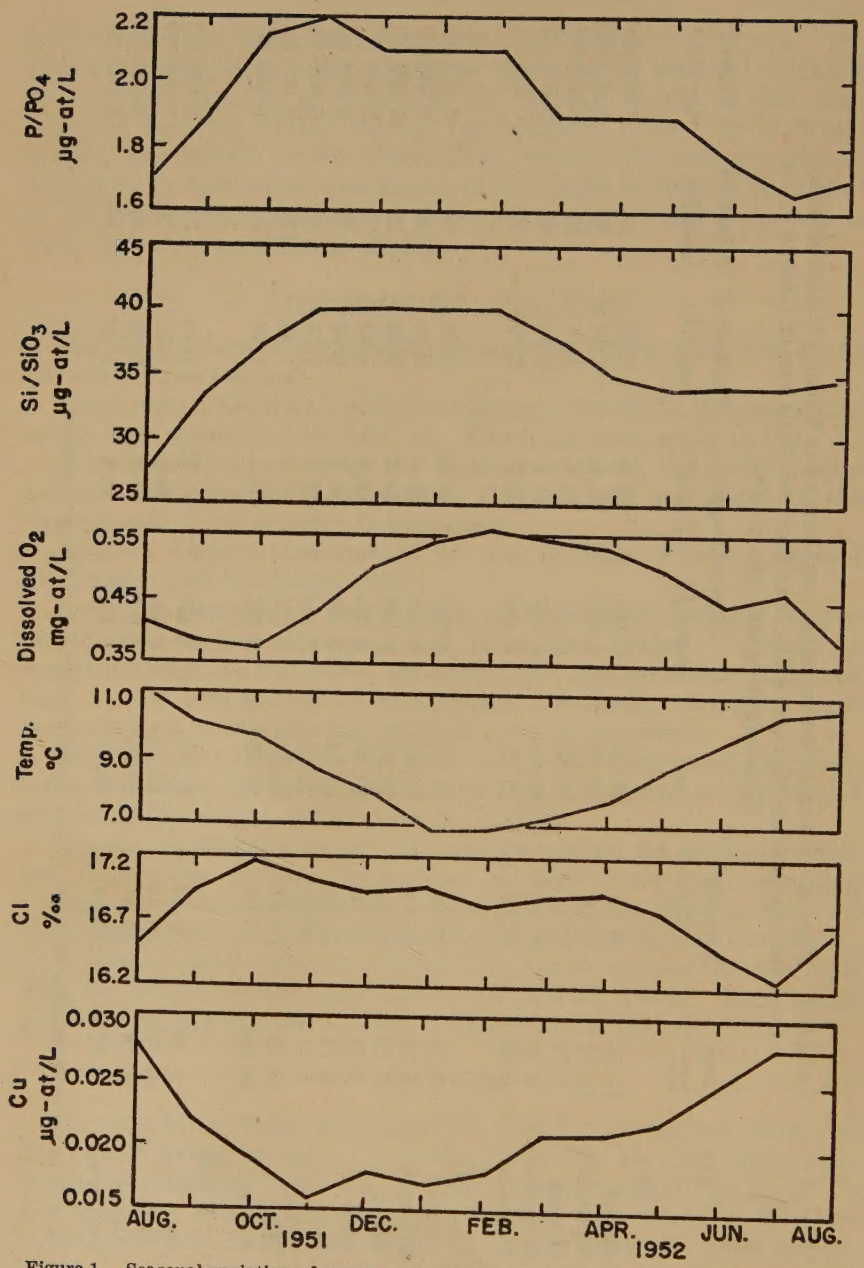

Figure 1. Seasonal variation of copper concentration and various oceanographic data for the surface waters of San Juan Channel, Washington.

The maximum chlorinity, 17.21 ‰ Cl (31.09 ‰ S), was observed in October, the minimum, 15.00 ‰ Cl (27.11 ‰ S), in mid-July. This minimum salinity occurred with the maximum temperature of 13.5° C.

The highest values for dissolved oxygen were obtained during the winter months, after which gradual decreases occurred until the lowest values were reached in October.

With the first bloom of diatoms in February, the concentrations of organic phosphates and silicates decreased and continued to decrease until the lowest values were reached in July. The highest concentrations, associated with the highest salinities of the year, were attained in November.

The monthly average copper concentration of the surface waters showed a definite seasonal trend with an autumn minimum and a summer maximum. In other words, the concentration of copper varied inversely with those of phosphates and silicates. Increase in copper content became evident in March and continued until July, when the highest values were noted. From August to November the copper concentrations decreased until minimum values were obtained. From November to March the copper content remained rather constant.

On various occasions during the summer months, abnormally high values for copper were sometimes encountered in surface water samples, particularly in those taken at times of slack tide and zero wind. Values as high as 0.060 μg-at/L were noted on several occasions. Fluctuations in the copper concentration between individual samples appeared to be greatest during the summer months and occurred in waters of low salinities and high temperatures at times of lower low tides when there was no meteorological disturbance.

A station in San Juan Channel was occupied to ascertain the vertical distribution of copper in water by using the ZoBell sampling equipment. These data are shown in Table II. Several other vertical stations had been occupied prior to the use of the ZoBell sampler, but the data were considered unreliable because of probable contamination of the samples. The authors have found that any equipment made of bronze, brass, or copper alloys which has been coated, plated, or painted with protective substances is unreliable for taking samples on which copper is to be determined. Furthermore, they observed that care must also be exercised in securing surface samples free of contamination from a ship, especially if copper paint has been used on the hull.

The chemical composition of the inshore waters varies not only with the season but to a lesser extent with the tide. Observations made

TABLE II. Vertical Distribution of Copper in the Waters of San Juan Channel, Washington, July 25, 1952. Lat. 48° 36.4′ N; Long. 123° 04.4′ W.

Depth (m)	*Temp.* (°C)	*Chlorinity* (‰)	δ_t	*Copper* (μg-at/L)
0	10.4	16.76	23.22	0.020
10	10.4	16.76	23.22	0.020
20	10.3	16.77	23.25	0.021
40	10.1	16.78	23.30	0.022
70	10.1	16.91	23.50	0.022
100	10.0	16.92	23.52	0.020
120	9.7	16.95	23.60	—
140	9.6	16.96	23.64	—

during a tidal cycle showed a range of 0.013 to 0.018 μg-at/L for surface waters. Higher values were obtained on the ebbing tide.

For comparison, surface sea water samples were collected in the Pacific Ocean from 60 to 500 miles beyond the Washington coast as well as at the entrance of the Strait of Juan de Fuca. The results are given in Table III. The waters of several lakes in Washington—three in Seattle in King County, two in Snohomish County, and two on San Juan Island in San Juan County—were analyzed for the copper content; the results are given in Table IV.

TABLE IV. Concentration of Copper in Several Fresh Water Lakes Near Seattle and on San Juan Island, Washington

Location	Copper, μg-at/L
Lake Washington, Seattle, King County	0.065
Lake Union, Seattle, King County	0.070
Salmon Bay, Seattle, King County	0.075
Hall Lake, Snohomish County	0.080
Chase Lake, Snohomish County	0.085
Echo Lake, San Juan Island	0.090
Sportsman Lake, San Juan Island	0.105

DISCUSSION

The copper concentration in 200 samples of surface water collected in San Juan Channel ranged from 0.012 to 0.060 μg-at/L, with an average of 0.023. The data demonstrate a definite seasonal trend, with a summer monthly maximum of 0.028 μg-at/L and an autumn minimum of 0.016 μg-at/L. The concentration of phosphates and silicates, determined simultaneously with the copper, showed summer minima and winter maxima and thus varied inversely as the copper. The

TABLE III. Copper and Various Oceanographic Data for the Surface Waters of the Pacific Ocean

Date	Location	Temp. (°C)	Chlorinity (‰)	σt	Dissolved Oxygen (mg-at/L)	Dissolved Oxygen (% Sat'n)	Copper (μg-at/L)
Aug. 19, 1952	Lat. 48° 30′ N Long. 124° 44′ W	10.86	17.38	24.02	0.435	76.5	0.018
Aug. 19, 1952	Lat. 48° 31′ N Long. 126° 47′ W	—	17.97	—	0.545	—	0.020
Aug. 20, 1952	Lat. 48° 33′ N Long. 130° 04′ W	15.69	17.99	23.92	—	—	0.016
Aug. 23, 1952	Lat. 47° 37′ N Long. 136° 15′ W	14.93	18.05	24.17	0.540	103.3	0.018
Aug. 25, 1952	Lat. 46° 52′ N Long. 128° 27′ W	14.91	18.00	24.10	0.525	100.4	0.020
Aug. 29, 1952	Lat. 46° 45′ N Long. 125° 40′ W	13.76	17.82	24.09	—	—	0.016

inverse relationship of copper to phosphates, silicates and density of the waters is shown in Fig. 2.

Atkins (1953), in studies of copper concentration for the waters of the English Channel, obtained a maximum of 0.39 μg-at/L during the

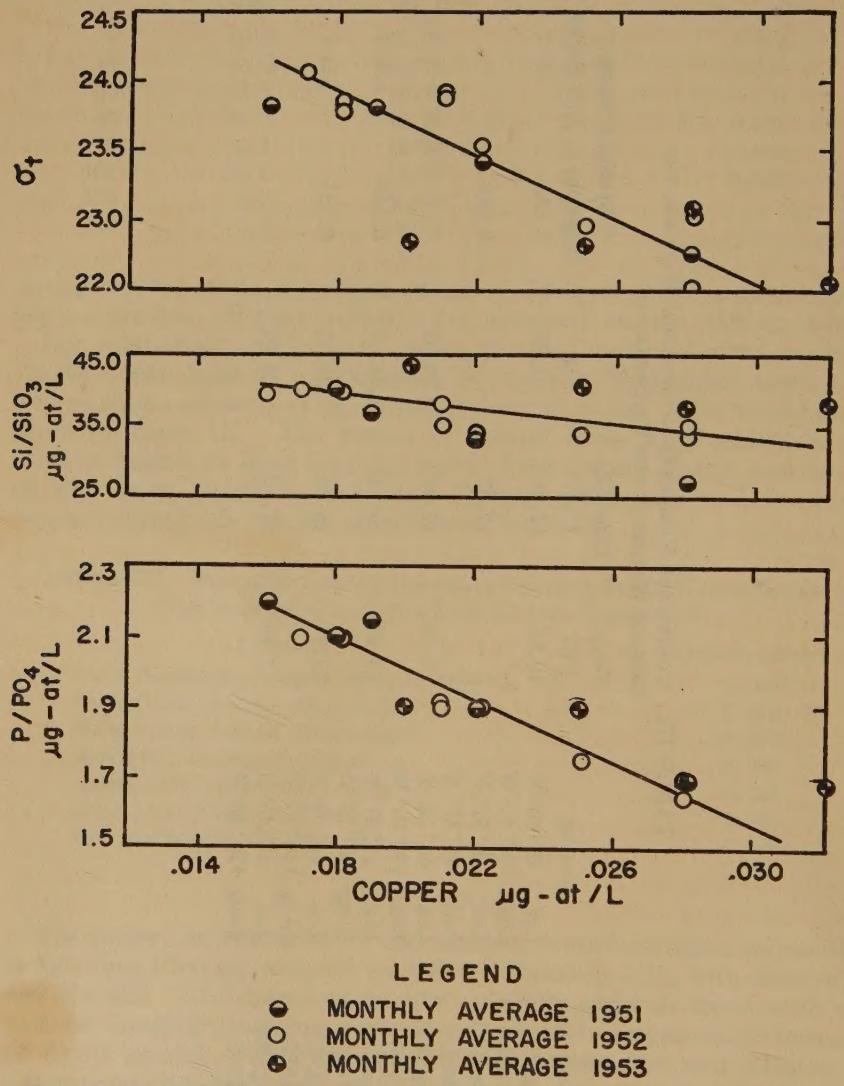

Figure 2. Concentration of copper related to values of phosphates, silicates and density of the surface waters of San Juan Channel, Washington.

winter months and a minimum of 0.024 μg-at/L during the autumn. His minimum value agrees well with our over-all average, but his maxima are much greater than those obtained for the waters of San Juan Channel. The seasonal trend of copper for the English Channel is just the reverse of that observed by us.

The results obtained by Morita (1950, 1953) for copper concentrations in waters of Tokyo Bay and Ise Bay, Japan, are closer in agreement with our data. He found a higher concentration, 0.038 μg-at/L, near shore and attributed this to land drainage. The concentration of copper decreased as the open ocean was approached until values of 0.012 μg-at/L were obtained for the surface waters of the Kuroshio.

The highest observed concentrations of copper in San Juan Channel were found in waters of low salinity during the summer months and at times of lower low tide. The low salinity waters of the region investigated resulted from flood waters of the Fraser River in Canada, which is by far the largest river of the region. The river discharges into the Strait of Georgia, where marked salinity and temperature gradients are noted, particularly during and immediately after flood stages. Favorable winds and tidal currents carry some of the warm and diluted water of the upper strata into San Juan Channel during July and August (Phifer and Thompson, 1937).

Results of several spot checkings of the Fraser River, in unpublished data of the British Columbia Research Council of Canada (Wright, 1952), show that some of the upstream tributaries of the river contained copper in concentrations four times higher than the saline waters of the San Juan Islands area. The relatively greater concentration of copper indicated for the Fraser River may account to a partial degree for the increasing copper in the surface waters of San Juan Channel during the summer.

The data given in Tables I and III indicate that the waters of San Juan Channel have approximately the same copper concentration as those of the Pacific Ocean during a major portion of the year. The copper content given in Table IV for a few lakes in Washington and that quoted for the Fraser River are considerably higher than the copper content of the open ocean waters. From these observations it might appear logical to conclude that the relatively high copper concentrations in the San Juan Channel waters (see Table I) are the result of land drainage. This is undoubtedly true to a certain extent, but it cannot explain the abnormally high individual values of copper obtained occasionally during the early summer months.

Observations of a similar nature have been reported for the Mississippi River estuary (Riley, 1937) and for Long Island Sound (Prytherch, 1934; Galtsoff, 1943). The high values given for Long

Island Sound may be attributed in part to the discharge into the Sound of considerable industrial waste carried by rivers which pass through many cities engaged in the manufacture of brass and copper products.

The high values noted for San Juan Channel on several occasions during the summer months can hardly be attributed to river discharge or to industrial pollution. It is therefore postulated that hydrogen sulfide, produced in certain regions of the area where there are extensive mud flats or where marked anaerobic conditions exist, reacts with the soluble copper in the sea water to precipitate copper sulfide which thus accumulates in the muds. Furthermore, copper contained in decaying organisms is likewise precipitated. The low tides of the region in the summer months occur during the daylight hours, the heat of the sun partially dries the mud flats, and the increased temperature favors the oxidation of the insoluble copper sulfide to the readily soluble copper sulfate. The drainage waters from these mud flats, warmer than the adjacent water, will have a high copper concentration, and, because of the lower density, will tend to float as surface water. It is also postulated that precipitation of copper as copper sulfide is favored during the colder months of the year when the oxygen content of the bottom waters is less than that during the summer months and when conditions for oxidation in the mud flats are less favorable for oxidation.

To partially demonstrate the correctness of these assumptions, East Sound, a body of water that extends into the southern side of Orcas Island in the San Juan Archipelago, was selected because there is practically no drainage from the adjacent land during the summer months and no industrial pollution. It is 11 km in length and has an average width of 2 km. As the central portion of the Sound is approached, the width is constricted for a short distance to 1 km; the head of the Sound widens to 3 km. There is no threshold or shallow ridge at the entrance to the Sound. The depth throughout is rather uniform from 25 to 30 m. As the head of the Sound is approached, shoaling occurs so that extensive mud flats lie exposed at low tides, but otherwise the shores are rather rocky and steep. The tidal range is about 2 m, but there is little tidal turbulence except in the constricted portion of the Sound. The waters are characterized by an unusually rich diatom growth in the spring and early summer months. Interchange with outside water is comparatively slow. The bottom, largely mud, is rich in organic matter. From the region of constricted width to the head of the Sound, the muds from the bottom contain quantities of hydrogen sulfide which can be detected only in the waters secured within the first few centimeters from the bottom.

Preliminary studies showed that the concentration of copper in the surface waters tended to increase toward the head of the Sound, as is indicated in the summary of data taken during the months of two summers on six different occasions.

Location	*No. of Samples*	*Average Cu Values* (μg-at/L)	*Minimum and Maximum Cu Values* (μg-at/L)
Area beyond East Sound	6	0.027	0.018–0.041
Entrance to East Sound	5	.038	.021–0.060
Constricted part of East Sound	9	.034	.024– .065
Head of East Sound, various parts	14	.046	.024– .132

In order to ascertain the amount of copper in the sediments, short core samples were secured in polyethylene tubes inserted in the core samplers. The water immediately above the sediment in the tubes was analyzed for copper and also for soluble inorganic phosphates. The top portions of the sediments in the cores were analyzed for copper with the following results:

Location	*—Analysis of Water—* PO_4/P μg-at/L	*Cu* μg-at/L	*Analysis of Sediments Cu* μg-at/kg
Area beyond East Sound	2.2	0.17	382
Entrance to East Sound	2.2	.33	457
Constricted Part of Sound	1.8	.26	502
Head of East Sound (5 different samples)	5.4	.34	590

A hole was dug in the mud flat at the head of the Sound, and analysis of the water from it showed 0.86 μg-at/L Cu and 8.0 μg-at/L inorganic phosphate-phosphorus. The waters in several tide pools showed an average of 0.050 μg-at/L Cu while the concentration of copper in the sea water 3 meters from the shore was 0.042 μg-at/L. All of these results indicate potentialities for the enrichment of surface waters with copper and phosphates that result from reactions in the intertidal area. The degree of this enrichment may be greater for copper than for phosphates, because, if it is assumed that 1 liter of drainage water from a mud flat containing 0.86 μg-at/L Cu and 8.00 μg-at $PO_4/P/L$ mixes with 100 liters of adjacent sea water containing 0.024 μg-at/L Cu and 1.0 μg-at $PO_4/P/L$, the copper concentration in the resulting mixture would have an increase of well over 30% while the phosphates would increase less than 10%.

Thus it seems possible that the occasionally high copper encountered in the surface waters of San Juan Channel may be due not only to river discharge but to conditions that occur in shoals of the area, as exemplified by the conditions demonstrated for East Sound.

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SUBSTANDARD REFERENCE SOLUTIONS IN CHLORINITY DETERMINATIONS BY THE KNUDSEN METHOD[1]

By

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ABSTRACT

It is shown that natural sea water of any chlorinity may be used as a reference solution in the Knudsen method. The proper correction to apply to burette readings of samples can be obtained by means of the fundamental stoichiometric relationship used in computing the "Table of Titration." A table of values for the construction of graphs from which the corrections can be obtained and a sample graph are included.

In order to conserve the usually limited quantities of Copenhagen Normal Sea Water available to oceanographic laboratories in this country, it has been customary to use "substandard sea water" of known chlorinity as the reference solution in chlorinity determinations by the Knudsen method (Oxner, 1920). Substandard water generally consists of sea water whose concentration has been adjusted by dilution with distilled water or by addition of sodium chloride to bring it approximately to the chlorinity of Normal Sea Water (U. S. Navy Hydrographic Office, 1951). The source of the substandard sea water and the method of adjustment vary from laboratory to laboratory, but in all cases substandard water is made up in large quantities and its chlorinity is carefully determined by means of Normal Sea Water.

The practice of bringing the chlorinity of substandard water to a value that is approximately equal to that of Normal water is necessary because the "Table of Titration" in Knudsen's "Hydrographical Tables" (1901) is based on a reference solution of chlorinity 19.38 ‰ and therefore it can be used only with standard water in the range 19.31 to 19.45 ‰. The addition of substantial amounts of sodium chloride to natural sea water is not a desirable procedure, since it may

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introduce slight errors in the chlorinity determinations when titration is carried on under extremely high or low temperature conditions. Such errors stem from the facts that the densities of both sample and reference solution enter into the final result and that the coefficient of thermal expansion of a pure sodium choloride solution differs from that of natural sea water on which Knudsen's table of titration is based.

The purpose of this paper is to show that natural sea water of any chlorinity can be used as a reference solution in the Knudsen method. Such flexibility in the concentration of the substandard water not only eliminates the tedious adjustment of natural sea water to a chlorinity of 19.38 ‰ but actually opens the way for improvement in titration when dealing with water samples of low chlorinity. When the chlorinities of the sample and standard differ by many per mille, the standardization of titration techniques becomes difficult. These difficulties can be circumvented by using a substandard within the chlorinity range of the samples.

The keystone of the Knudsen method is the adjustment of the silver nitrate concentration so that at the end of the titration the number representing the burette reading is approximately equal to the chlorinity in per mille. A conventional analytical burette may be used for the titration so long as the burette reading in milliliters is divided by two and the concentration of silver nitrate is as specified by Knudsen. Knudsen's "Table of Titration" lists values of k, the correction to be added to the burette reading of the sample in order to convert it to chlorinity. The term k is a function of a (the burette reading of the sample) and α (the difference between the chlorinity and burette reading of Normal Sea Water). Its derivation from fundamental stoichiometric relationships is given in Knudsen, et al. (1902).

When a reference solution with a chlorinity other than 19.38 ‰ is used, the problem of determining the proper correction to the burette reading can be approached by two methods. In the first method, an auxiliary table or graph may be constructed for conversion of the difference between the chlorinity and the observed burette readings of the substandard to the equivalent value of α which would have been obtained if Normal Sea Water had been used. Such a graph or table is not difficult to construct from the fundamental relationship. However, it would apply to one chlorinity only, so that a new auxiliary graph or table would have to be constructed each time the reference solution was changed. A somewhat similar method has been discussed by Toll, *et al.* (1951).

The second approach consists of a general solution to the problem without reference to Knudsen's table. Such a solution would be extremely cumbersome in tabular form because of the large number of

arguments that would be required. However, it can readily be put into a compact graphical form.

Both of the above methods of determining k are derived from the basic stoichiometric equation

$$\frac{a_1}{Cl_1\,\rho_1} = \frac{a_2}{Cl_2\,\rho_2},$$

where Cl = chloride concentration in per mille; ρ = specific gravity (density in kg/l) of the sample; and a = burette reading (volume of $AgNO_3$). It is assumed that the volume and temperature of the samples are identical and that the same silver nitrate solution is used for their titration. Furthermore, following Knudsen's definition,

$$a_1 = Cl_1 - k_1, \qquad \text{and} \qquad a_2 = Cl_2 - k_2.$$

To develop an auxiliary table or graph to be used in conjunction with Knudsen's table, it is necessary to obtain Knudsen's α in terms of the burette reading of the substandard solution. In the basic equation let subscript $_1$ represent Normal Sea Water and subscript $_2$ substandard water. After making the substitutions, $Cl_1 = 19.38$ ‰ and $k_1 = \alpha$, the equation takes the form,

$$\frac{19.38 - \alpha}{19.38\,\rho_1} = \frac{a_2}{Cl_2\,\rho_2}.$$

Therefore,

$$\alpha = 19.38 - \frac{19.38\,\rho_1}{Cl_2\,\rho_2}\,a_2.$$

Since Cl_2 is a constant, i.e., the previously determined chlorinity of the substandard, and since ρ_1 and ρ_2 are unique functions of the respective chlorinities at any specified temperature (Knudsen used 17.5°C), it is apparent that α is a linear function of a_2, the burette reading of the substandard. The range of α covered by Knudsen's table extends from -0.150 to $+0.145$. If these two values are substituted in the above equation, the limits within which the burette reading of the substandard must lie in order to make this method applicable are readily determined.

The general solution is derived in the following manner. Under uniform conditions of titration, that is, when using a particular burette, volume of sample, concentration of silver nitrate, and constant temperature, the stoichiometric relationship between Cl, ρ, and a defines a constant

$$F = \frac{a_i}{Cl_i\,\rho_i},$$

where the subscript $_i$ refers to any sample or standard titrated under uniform conditions. The value of F is determined from the known chlorinity, the density, and the experimentally determined burette reading of the reference solution. Tables or graphs computed by this method can be used at any temperature normally encountered. A

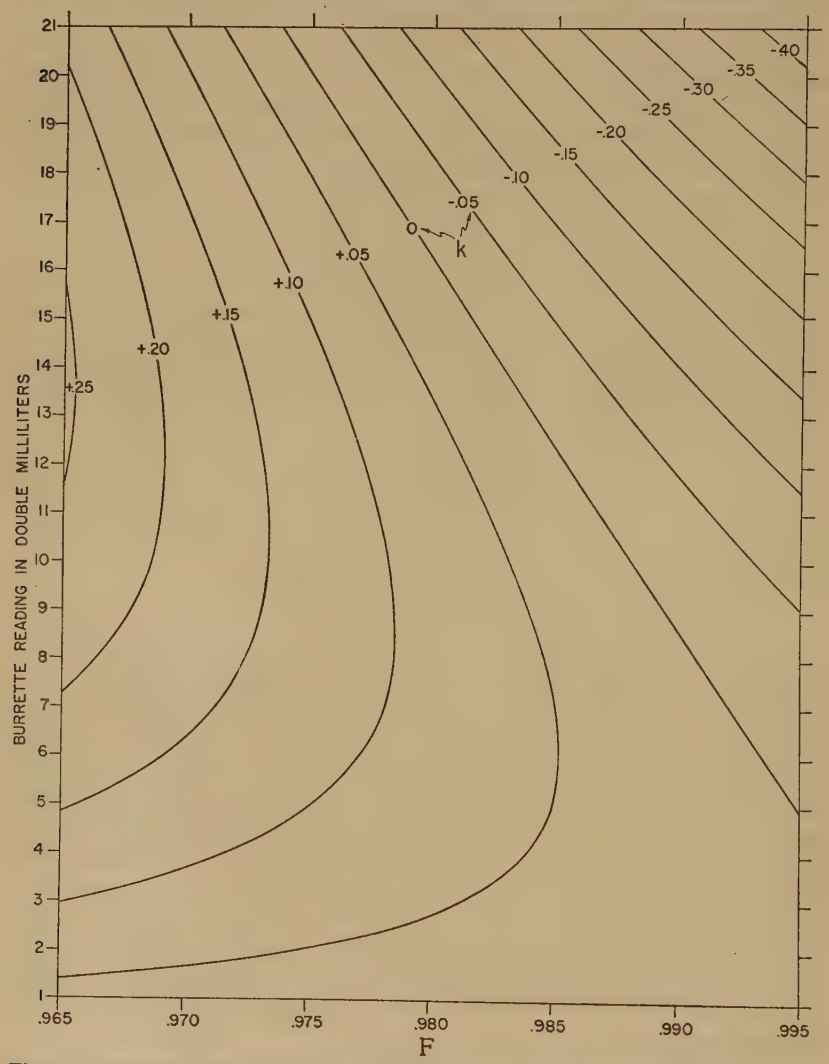

Figure 1. Example of a graph for finding k as a function of the parameter F and the burette reading of the sample. F is determined on the graph from the burette reading and the computed k of the substandard solution.

temperature change would only produce a different value of F for the reference solution.

A conversion graph based on the general solution requires F as a function of k_i and a_1. Since $Cl_i = a_i + k_i$ and Cl_i also determines ρ_i at a given temperature, a table of values for the preparation of such a graph can be computed by assuming pairs of values for Cl_i and a_i. Table I was prepared by this method using values of ρ at 20°C. The ranges of Cl_i and a_i were selected to permit the construction of graphs for burette readings between 1.00 and 21.00 and for values of F between 0.96500 and 0.99500. An example of such a graph, on a reduced scale, is shown in Figure 1.

In using the graphical method, a and k of the reference solution are determined by titration. The graph is then entered with these values to determine the F to be used for the series of samples. The actual value of F is irrelevant since it is merely the parametric curve on which all pairs of a and k values of the unknown samples lie for the particular concentration of silver nitrate and temperature. Thus the k for any sample is found by moving along the appropriate F curve to the intersection with the a curve of the sample in question.

TABLE I. F AS A FUNCTION OF k AND $(Cl \times \rho_{20})$

k \ $Cl‰$, ρ_{20}	1.00 0.99966	1.50 1.00035	2.00 1.00104	2.50 1.00172	3.00 1.00241	3.50 1.00309	4.00 1.00377	5.00 1.00514
+.20								
+.19								
+.18								.95907
+.17							.95390	.96106
+.16							.95639	.96305
+.15							.95888	.96504
+.14						.95704	.96138	.96703
+.13						.95989	.96387	.96902
+.12					.95769	.96274	.96636	.97101
+.11					.96102	.96559	.96885	.97300
+.10				.95835	.96434	.96844	.97134	.97499
+.09				.96234	.96767	.97128	.97383	.97698
+.08			.95900	.96634	.97099	.97413	.97632	.97897
+.07			.96400	.97033	.97432	.97698	.97881	.98096
+.06		.95966	.96899	.97432	.97764	.97983	.98130	.98295
+.05	.95032	.96633	.97399	.97832	.98097	.98268	.98379	.98494
+.04	.96033	.97299	.97898	.98231	.98429	.98553	.98628	.98693
+.03	.97033	.97966	.98398	.98630	.98762	.98837	.98877	.98892
+.02	.98033	.98632	.98897	.99030	.99095	.99122	.99126	.99091
+.01	.99034	.99299	.99397	.99429	.99427	.99407	.99375	.99290
.00	1.00034	.99965	.99896	.99828	.99760	.99692	.99624	.99489
−.01	1.01034	1.00631	1.00396	1.00228	1.00092	.99977	.99873	.99688
−.02						1.00262	1.00123	.99887
−.03								1.00086

TABLE I (CONTINUED)

$Cl‰$ / ρ_{20} / k	6.00 1.00651	7.00 1.00787	8.00 1.00924	9.00 1.01060	10.00 1.01196	11.00 1.01333	12.00 1.01469	13.00 1.01605
+.25			.95988	.96202	.96348	.96442	.96499	.96528
+.24			.96112	.96312	.96446	.96531	.96581	.96603
+.23			.96236	.96422	.96545	.96621	.96663	.96679
+.22			.96360	.96532	.96644	.96711	.96745	.96755
+.21			.96483	.96642	.96743	.96801	.96828	.96830
+.20		.96384	.96607	.96752	.96842	.96890	.96910.	.96906
+.19	.96207	.96526	.96731	.96862	.96941	.96980	.96992	.96982
+.18	.96373	.96668	.96855	.96972	.97039	.97070	.97074	.97058
+.17	.96538	.96810	.96979	.97082	.97138	.97159	.97156	.97133
+.16	.96704	.96951	.97103	.97192	.97237	.97249	.97238	.97209
+.15	.96869	.97093	.97227	.97302	.97336	.97339	.97320	.97285
+.14	.97035	.97235	.97350	.97412	.97435	.97429	.97402	.97360
+.13	.97201	.97376	.97474	.97522	.97533	.97518	.97485	.97436
+.12	.97366	.97518	.97598	.97632	.97632	.97608	.97567	.97512
+.11	.97532	.97660	.97722	.97742	.97731	.97698	.97649	.97588
+.10	.97697	.97802	.97846	.97852	.97830	.97787	.97731	.97663
+.09	.97863	.97943	.97970	.97962	.97929	.97877	.97813	.97739
+.08	.98028	.98085	.98094	.98072	.98028	.97967	.97895	.97815
+.07	.98194	.98227	.98217	.98181	.98126	.98057	.97977	.97890
+.06	.98360	.98369	.98341	.98291	.98225	.98146	.98059	.97966
+.05	.98525	.98510	.98465	.98401	.98324	.98236	.98142	.98042
+.04	.98691	.98652	.98589	.98511	.98423	.98326	.98224	.98118
+.03	.98856	.98794	.98713	.98621	.98522	.98415	.98306	.98193
+.02	.99022	.98936	.98837	.98731	.98620	.98505	.98388	.98269
+.01	.99188	.99077	.98961	.98841	.98719	.98595	.98470	.98345
.00	.99353	.99219	.99084	.98951	.98818	.98685	.98552	.98420
−.01	.99519	.99361	.99208	.99061	.98917	.98774	.98634	.98496
−.02	.99684	.99503	.99332	.99171	.99016	.98864	.98717	.98572
−.03	.99850	.99644	.99456	.99281	.99115	.98954	.98799	.98647
−.04	1.00016	.99786	.99580	.99391	.99213	.99043	.98881	.98723
−.05		.99928	.99704	.99501	.99312	.99133	.98963	.98799
−.06		1.00070	.99828	.99611	.99411	.99223	.99045	.98875
−.07			.99951	.99721	.99510	.99313	.99127	.98950
−.08			1.00075	.99831	.99609	.99402	.99209	.99026
−.09			1.00199	.99941	.99707	.99492	.99291	.99101
−.10			1.00323	1.00051	.99806	.99582	.99374	.99177
−.11			1.00447	1.00161	.99905	.99671	.99456	.99253
−.12					1.00004	.99761	.99538	.99329
−.13						.99851	.99620	.99405
−.14						.99941	.99702	.99480
−.15						1.00030	.99784	.99556

TABLE I (CONTINUED)

k \ $Cl‰$, ρ_{20}	14.00 1.01742	15.00 1.01879	16.00 1.02015	17.00 1.02153	18.00 1.02290	19.00 1.02427	20.00 1.02564	21.00 1.02701
+.25	.96533	.96520	.96493	.96453	.96403	.96346	.96281	.96211
+.24	.96603	.96585	.96554	.96510	.96458	.96397	.96330	.96257
+.23	.96673	.96651	.96616	.96568	.96512	.96449	.96379	.96304
+.22	.96743	.96716	.96677	.96626	.96567	.96500	.96428	.96350
+.21	.96814	.96781	.96738	.96683	.96621	.96551	.96476	.96396
+.20	.96884	.96847	.96799	.96741	.96675	.96603	.96525	.96443
+.19	.96954	.96912	.96861	.96798	.96729	.96654	.96574	.96489
+.18	.97024	.96978	.96922	.96856	.96784	.96706	.96623	.96536
+.17	.97094	.97043	.96983	.96913	.96838	.96757	.96671	.96582
+.16	.97165	.97109	.97045	.96971	.96892	.96808	.96720	.96628
+.15	.97235	.97174	.97106	.97029	.96947	.96860	.96769	.96675
+.14	.97305	.97240	.97167	.97086	.97001	.96911	.96818	.96721
+.13	.97375	.97305	.97228	.97144	.97055	.96962	.96866	.96768
+.12	.97445	.97370	.97290	.97210	.97110	.97014	.96915	.96814
+.11	.97516	.97436	.97351	.97259	.97164	.97065	.96964	.96860
+.10	.97586	.97501	.97412	.97317	.97218	.97117	.97013	.96907
+.09	.97656	.97567	.97473	.97374	.97272	.97168	.97061	.96953
+.08	.97726	.97632	.97535	.97432	.97327	.97219	.97110	.96999
+.07	.97796	.97698	.97596	.97489	.97381	.97271	.97159	.97046
+.06	.97867	.97763	.97657	.97547	.97435	.97322	.97208	.97092
+.05	.97937	.97828	.97718	.97604	.97490	.97374	.97256	.97139
+.04	.98007	.97894	.97780	.97662	.97544	.97425	.97305	.97185
+.03	.98077	.97959	.97841	.97720	.97598	.97476	.97354	.97231
+.02	.98147	.98025	.97902	.97777	.97653	.97528	.97403	.97278
+.01	.98218	.98090	.97964	.97835	.97707	.97579	.97451	.97324
.00	.98288	.98156	.98025	.97892	.97761	.97630	.97500	.97370
−.01	.98358	.98221	.98086	.97950	.97816	.97682	.97549	.97417
−.02	.98428	.98287	.98147	.98008	.97870	.97733	.97598	.97463
−.03	.98498	.98352	.98209	.98065	.97924	.97785	.97646	.97509
−.04	.98569	.97417	.98270	.98123	.97979	.97836	.97695	.97556
−.05	.98639	.98483	.98331	.98180	.98033	.97887	.97744	.97602
−.06	.98709	.98548	.98392	.98238	.98087	.97939	.97793	.97649
−.07	.98779	.98614	.98454	.98295	.98141	.97990	.97841	.97695
−.08	.98849	.98679	.98515	.98353	.98196	.98042	.97890	.97741
−.09	.98920	.98745	.98576	.98411	.98250	.98093	.97939	.97788
−.10	.98990	.98810	.98637	.98468	.98304	.98144	.97988	.97834
−.11	.99060	.98875	.98699	.98526	.98359	.98196	.98036	.97880
−.12	.99130	.98941	.98760	.98583	.98413	.98247	.98085	.97927
−.13	.99200	.99006	.98821	.98641	.98467	.98298	.98134	.97973
−.14	.99271	.99072	.98883	.98699	.98522	.98350	.98183	.98019
−.15	.99341	.99137	.98944	.98756	.98576	.98401	.98231	.98066

TABLE I (CONTINUED)

$Cl^0/_{00}$ / ρ_{20} / k	14.00 1.01742	15.00 1.01879	16.00 1.02015	17.00 1.02153	18.00 1.02290	19.00 1.02427	20.00 1.02564	21.00 1.02701
−.16	.99411	.99203	.99005	.98814	.98630	.98453	.98280	.98112
−.17	.99481	.99268	.99066	.98871	.98685	.98504	.98329	.98159
−.18	.99552	.99334	.99128	.98929	.98739	.98555	.98378	.98205
−.19	.99622	.99399	.99189	.98986	.98793	.98607	.98426	.98251
−.20	.99692	.99464	.99250	.99044	.98847	.98658	.98475	.98298
−.21	.99762	.99530	.99311	.99102	.98902	.98710	.98524	.98344
−.22	.99832	.99595	.99373	.99159	.98956	.98761	.98573	.98390
−.23	.99903	.99661	.99434	.99217	.99010	.98812	.98621	.98437
−.24	.99973	.99726	.99495	.99274	.99065	.98864	.98670	.98483
−.25	1.00043	.99792	.99556	.99332	.99119	.98915	.98719	.98530
−.26		.99856	.99618	.99390	.99173	.98966	.98768	.98576
−.27		.99922	.99679	.99447	.99228	.99018	.98816	.98622
−.28		.99988	.99740	.99505	.99282	.99069	.98865	.98669
−.29		1.00053	.99801	.99562	.99336	.99121	.98914	.98715
−.30		1.00119	.99863	.99620	.99391	.99172	.98963	.98761
−.31			.99924	.99677	.99445	.99223	.99011	.98808
−.32			.99985	.99735	.99499	.99275	.99060	.98854
−.33			1.00047	.99793	.99554	.99326	.99109	.98900
−.34			1.00108	.99850	.99608	.99378	.99158	.98947
−.35			1.00169	.99908	.99662	.99429	.99206	.98993
−.36				.99965	.99716	.99480	.99255	.99040
−.37				1.00023	.99771	.99532	.99304	.99086
−.38				1.00081	.99825	.99583	.99353	.99132
−.39				1.00138	.99879	.99634	.99401	.99179
−.40				1.00196	.99934	.99686	.99450	.99225
−.41					.99988	.99737	.99499	.99271
−.42					1.00042	.99789	.99548	.99318
−.43					1.00097	.99840	.99596	.99364
−.44					1.00151	.99891	.99645	.99410
−.45					1.00205	.99943	.99694	.99457
−.46						.99994	.99743	.99503
−.47						1.00046	.99791	.99550
−.48						1.00097	.99840	.99596
−.49						1.00148	.99889	.99642
−.50						1.00200	.99938	.99689
−.51							.99986	.99735
−.52							1.00035	.99781
−.53							1.00084	.99828

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STEADY FLOW IN A FRICTIONLESS HOMOGENEOUS OCEAN[1]

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ABSTRACT

A mathematical model is developed to study the free (frictionless) steady horizontal flow which can occur in a homogeneous ocean of constant depth. The flow satisfies the dynamic constraint that the vertical component of absolute vorticity is constant along a streamline. The conclusion is reached that in an enclosed ocean a free steady circulation cannot have any slow broad eastward currents. The eastward currents must occur as narrow streams of high velocity and high relative vorticity. Intensified currents are present along the eastern and western coasts.

The theory which is developed for the homogeneous ocean of constant depth can be applied to the two-layer ocean if the horizontal divergence of flow is negligible. If the horizontal divergence is not negligible, then the intensification of poleward currents is more pronounced and that of equatorward currents less pronounced as compared with the homogeneous ocean.

INTRODUCTION

The theoretical models of the wind-induced oceanic circulation developed by Stommel (1948), Munk (1950), and others have contributed a great deal to the understanding of some large-scale features of the circulation in the major oceans, but these models are limited by the fact that they are based on linearized equations and on the artificial concept of eddy viscosity. It is possible that significant features of oceanic circulation cannot be simulated by a model in which the nonlinear terms, i. e., the relative-acceleration terms, are neglected. Attempts to improve and extend these models are hindered by the analytical complexity of a model which includes both relative accelerations and dissipative effects.

In the present paper, simple types of ideal fluid motion are studied in order to describe characteristic features of steady free circulation in an enclosed ocean. By considering an ideal fluid, the analytical difficulties presented by the nonlinear terms in the hydrodynamic equations

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are avoided. It is possible to show that some features of oceanic circulation, e.g., the westward intensification of the circulation, exist in the absence of friction.

The simplest model that includes accelerations in the flow is a frictionless homogeneous constant-depth ocean in which the water moves horizontally in the absence of driving forces. This model does not contain any mechanism by which free flow can be established from a state of rest, but it does contain certain dynamic restrictions which must be satisfied by an existing free flow. For steady flow, the most important constraint is that the vertical component of absolute vorticity is constant along a streamline. This constraint on the absolute vorticity is stringent, because the change of the Coriolis parameter over a few degrees of latitude is, over most of the real ocean, about two orders of magnitude greater than the relative vorticity of the flow. It is apparent, therefore, that the possible types of free flow are severely restricted in the ocean.

Rossby (1936) has considered the dynamic constraint on the vertical component of absolute vorticity in his study of the Gulf Stream, but his analysis did not extend to the entire ocean. In the study of free atmospheric flow, there has been more progress and some attempts have been made to adapt the results of these studies to circulation in the oceans. Høiland (1950) found particular examples of free flow which he applied to the ocean. Although these examples agree qualitatively with the free flows discussed in the present paper, Høiland fails to point out the pronounced characteristics that the free flow would exhibit.

ASSUMPTIONS AND NOTATION

The ocean model chosen in this analysis consists of a homogeneous body of water of constant depth bounded by smooth vertical lateral boundaries. The motion is assumed to be horizontal and uniform with depth. The effects of thermohaline structure, compressibility, and frictional forces are neglected everywhere. Vertical motions and accelerations are also neglected so that the pressure is simply determined by the hydrostatic equation.

The unit vector $\mathbf{k}$ is directed vertically upward, and z is the vertical distance measured upward from a level undisturbed ocean surface. The ocean surface in the model is at $z = \eta$. The horizontal velocity is designated by $\mathbf{v}$ and the stream function of the horizontal circulation by ψ. The vertical component of absolute vorticity ζ_a equals $f + \zeta$, where f is the Coriolis parameter and ζ is the vertical component of relative vorticity. The quantity Q is introduced for $p/\rho + \frac{1}{2}(\mathbf{v}\cdot\mathbf{v}) + gz$, where p is pressure, ρ density, and g acceleration of gravity.

EQUATIONS OF MOTION

The equations of motion, consistent with the assumptions made above, may be written

$$(\mathbf{v}\cdot\nabla)\,\mathbf{v} + f(\mathbf{k}\times\mathbf{v}) = -\nabla\, p/\rho\,, \tag{1}$$

$$\nabla\cdot\mathbf{v} = 0\,, \tag{2}$$

$$p = \rho g(\eta - z)\,, \tag{3}$$

where the differential vector operator ∇ refers to the horizontal coordinates only.

Equation (1) may be written $\zeta_a(\mathbf{k}\times\mathbf{v}) = -\nabla Q$ or

$$\zeta_a\mathbf{v} = -\nabla\times(Q\mathbf{k})\,, \tag{4}$$

where

$$Q = g\eta + \tfrac{1}{2}(\mathbf{v}\cdot\mathbf{v})\,. \tag{5}$$

The velocity stream function ψ is defined by the equation

$$\mathbf{v} = \nabla\times(\psi\mathbf{k})\,. \tag{6}$$

Elimination of $\mathbf{v}$ from equation (4) yields

$$\zeta_a\nabla\psi = -\nabla Q\,, \tag{7}$$

so that

$$Q = Q(\psi)\,, \tag{8}$$

$$\zeta_a = -\,dQ/d\psi\,. \tag{9}$$

Equation (9) is a statement of the well known result that the vertical component of absolute vorticity is constant along a streamline. This equation expresses the basic dynamic constraint to be satisfied by the steady free motion in a homogeneous ocean.

DIMENSIONAL ANALYSIS

The following dimensional constants are chosen as being representative of a typical ocean:

L_0 = 2000 km, a typical horizontal length;
f_0 = 10^{-4}, a typical value of the Coriolis parameter,
η_0 = 100 cm, a typical displacement of the free surface from the undisturbed level $z = 0$;
g = 10^3 cm sec^{-2}, gravity.

The equations are converted into nondimensional form by introducing the nondimensional variables $\mathbf{v}' = \mathbf{v}/U$, where U is a charac-

teristic velocity, $\psi' = \psi/UL_0$, $Q' = Q/g_0$, $\zeta_a' = \zeta_a/f_0$, $f' = f/f'_0$, $\zeta' = \zeta/(U/L_0)$, and $\nabla' = L_0\nabla$, the nondimensional differential operator.

By choosing $U = g\eta_0/f_0L_0 = 5$ cm sec^{-1} and defining $\delta = U/f_0L_0 = 2.5 \times 10^{-4}$, the nondimensional form of (6), (8), and (9) may be written

$$\mathbf{v}' = \nabla' \times (\psi'\mathbf{k}) , \tag{10}$$

$$Q' = Q'(\psi') = \eta' + \tfrac{1}{2}\,\delta(\mathbf{v}'\cdot\mathbf{v}') , \tag{11}$$

$$\zeta_a' = -\,dQ'/d\psi' = f' + \delta\zeta' = f' - \delta\nabla'^2\psi' . \tag{12}$$

THE VORTICITY EQUATION

The vorticity equation (12) contains two unknown functions, ψ' and ζ_a', whereas (11) contains three unknown functions, ψ', Q', and η'. Therefore the motion is restricted only by (12), because η' can always be chosen to satisfy (11) once ψ' and ζ_a' are determined. Equation (12) does not determine the motion uniquely, because the vertical component of absolute vorticity can be any function of the stream function.

It is evident from (12) that, unless relative vorticities comparable with the Coriolis parameter (i.e., $\zeta' \sim 1/\delta$) are present in the flow, the stream function will be a function of f' only and the flow will be along latitude circles only. It follows, therefore, that in the vicinity of meridional boundaries where the flow must be across latitude circles, the relative vorticity and hence the velocity must be much larger than the average for the ocean. It is not possible to have in an enclosed ocean a steady free circulation which does not contain relatively large velocities unless the circulation is confined to a narrow range of latitude. For certain choices of $\zeta_a'(\psi')$, the circulation cannot be slow, e.g., ζ_a' = constant. In order to obtain a steady free circulation that is slow over most of the ocean, restrictions must be imposed upon the choice of $\zeta_a'(\psi')$. The nature of these restrictions is illustrated by considering a simple example.

A SIMPLE EXAMPLE OF FREE FLOW

A rectangular nondimensional coordinate system x', y' is introduced with the positive x'-axis eastward and the positive y'-axis northward. A rectangular region with sides a', b' is chosen to represent the ocean. The function $\zeta_a'(\psi')$ is assumed to be a linear function of ψ', so that (12) becomes

$$f' - \delta\nabla'^2\psi' = c_0 + c_1\psi' , \tag{13}$$

where c_0 and c_1 are constants.

The boundary condition on ψ' is that the ocean boundary is a

streamline. This condition is satisfied by requiring the stream function to be zero along the ocean boundary.

The Coriolis parameter is assumed to be a linear function of y' of the form $f' = f_0' + \beta y'$.

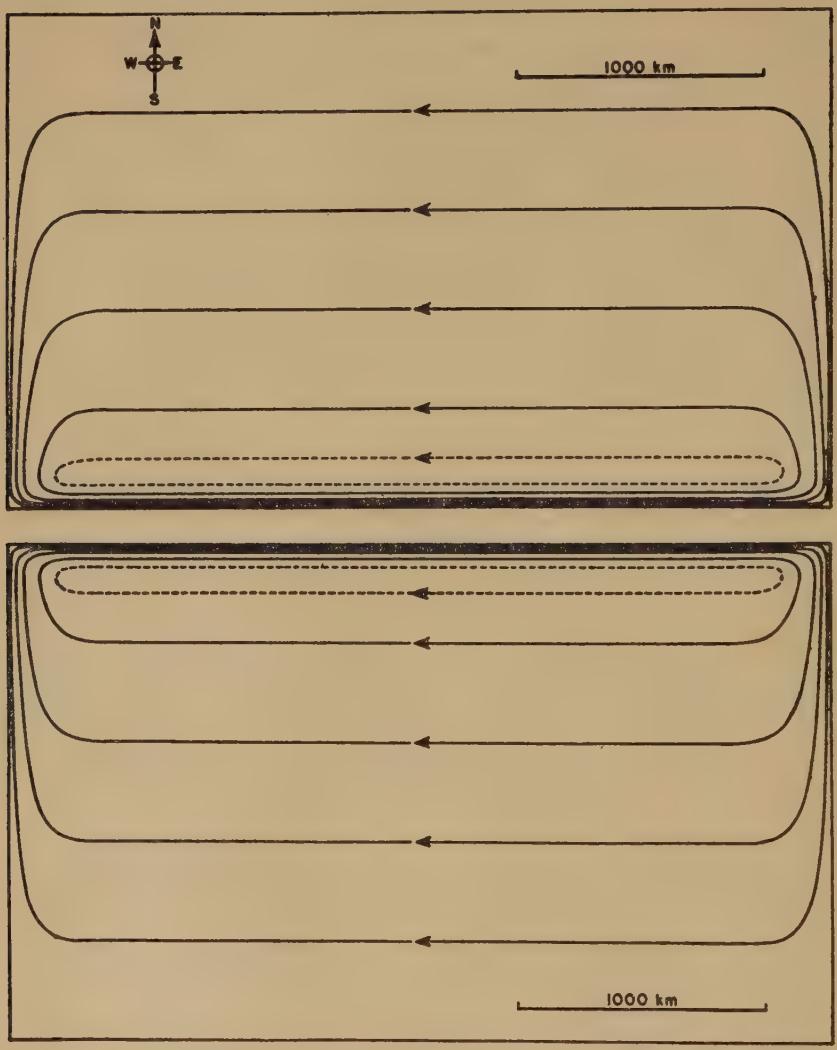

Figure 1. Streamlines of free flow in a homogeneous ocean, computed from equation (15) for $y_0' = b'$, $b' = 1$ in the upper diagram and for $y_0' = 0$, $b' = 1$ in the lower diagram. The number b' is the ratio of the meridional extent of the ocean to L_0.

By defining u_0' and y_0' so that $u_0' = \beta/c_1$ and $y_0' = (c_0 - f_0')/c_1$, equation (13) may be written

$$(u_0'\delta/\beta)\nabla'^2\psi' + \psi' = u_0'(y' - y_0') . \qquad (14)$$

In equation (14), u_0' is a nondimensional velocity, δ a ratio of relative vorticity to the Coriolis parameter, and β the nondimensional northward variation of the Coriolis parameter.

Wherever the relative vorticity is negligible, the velocity is given by u_0'; eastward if u_0' is positive and westward if negative. A study of (14) reveals (a) that the relative vorticity in an enclosed ocean can be negligible only if u_0' is negative and (b) that all eastward currents

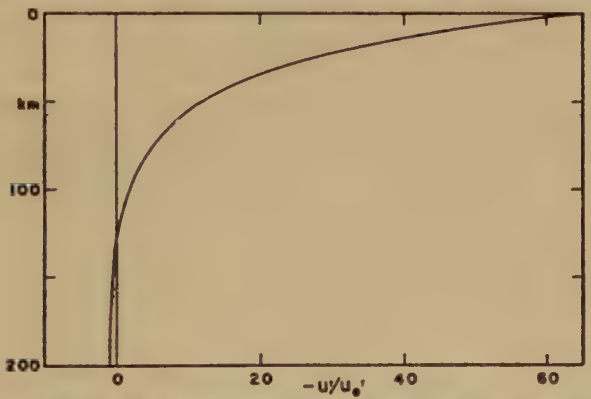

Figure 2. Velocity distribution across the eastward jet, computed for $y_0' = 0$, $b' = 1$, and $L_0 = 2000$ km. The number b' is the ratio of the meridional extent of the ocean to L_0.

must occur as narrow streams of high velocity and high relative vorticity.

Although the exact solution of (14) can be obtained, the simpler approximate solution obtained by boundary-layer methods is more convenient. The boundary-layer solution may be written

$$\psi' = u_0'\{(y' - y_0') + y_0'\exp(-y'/\varepsilon) - (b' - y_0')\exp[-(b' - y')/\varepsilon]\} \times \{1 - \exp(-x'/\varepsilon) - \exp[-(a' - x')/\varepsilon]\} , \qquad (15)$$

where $\varepsilon = (|u_0'\delta/\beta|)^{\frac{1}{2}}$. This solution is valid so long as $b'/\varepsilon \gg 1$ and $a'/\varepsilon \gg 1$.

In Fig. 1, streamlines of the free flow are shown for $y_0' = b'$, $b' = 1$ in the upper diagram and for $y_0' = 0$, $b' = 1$ in the lower diagram. These diagrams may be joined to give a symmetrical jet along the middle of the ocean, flanked by broad westward currents to the north

and south. The velocity distribution across the eastward jet for $y_0' = 0$, $b' = 1$ is computed from the equation

$$u'/u_0' = 1 - (1/\epsilon) \exp [- (1 - y')/\epsilon]$$

and is shown in Fig. 2. All the figures are based on computations for which $\epsilon^2 = 2.5 \times 10^{-4}$.

DISCUSSION

It is apparent from the preceding analysis that the free steady circulation in an enclosed ocean has distinctive features due to the variation of Coriolis parameter with latitude. The circulation, characterized by a slow westward drift over the major portion of the ocean, is accelerated and concentrated into a swift current along the western boundary of the ocean. The eastward current occurs as a narrow stream of high velocity and high relative vorticity. This eastward jet which crosses the ocean at constant speed is decelerated and spread out along the eastern boundary of the ocean. A slow, broad, eastward current cannot exist in the steady state.

The east-west symmetry of the stream function for the homogeneous ocean cannot exist if any dissipation is present. The swift current along the eastern boundary of the ocean, which must be supplied by a jet, is very sensitive to losses of relative vorticity in the jet.

The concentration of the westward drift into an eastward jet would occur in the real ocean if frictional stresses were insufficient to decelerate the current along the western boundary of the ocean. The presence of the Gulf Stream in the open ocean northeast of Cape Hatteras suggests that a considerable portion of the momentum and relative vorticity of the Stream actually reaches the open ocean and is dissipated there.

EXTENSION TO THE TWO-LAYER OCEAN

Most of the real ocean has a more or less clearly defined upper layer which contains the relatively strong motion and which represents the portion of the ocean influenced by wind stress. Below this layer there is a much larger body of water of greater density which shows little motion and little response to the prevailing atmospheric state. This picture of the ocean has often been represented in theoretical considerations, in idealized form, by the two-layer ocean consisting of two homogeneous layers of water of different densities with the motion confined entirely to the upper less dense layer.

The two-layer ocean in its undisturbed state may be completely specified by giving the lateral boundaries, the densities ρ_0 and ρ_1 of the upper and lower layers respectively, and the volume V_0 of the

upper layer. In order to adapt the analysis of the homogeneous ocean to the two-layer ocean, the additional quantities η_1, h, ψ, σ, γ are introduced and defined as follows: $z = \eta_1$, the equation for the interface between the two layers; $h = \eta + \eta_1$, the thickness of the upper layer; ψ, the stream function of volume transport; $\sigma = \nabla \cdot \mathbf{v}$, the divergence of the horizontal velocity; $\gamma = \rho_0/(\rho_1 - \rho_0)$.

The pressure in the lower layer is given by

$$p = \rho_0 g(\eta + \eta_1) - \rho_1 g(z + \eta_1) . \tag{16}$$

The condition for no motion in the lower layer requires the horizontal pressure gradient to be zero everywhere in the lower layer, so that

$$\begin{aligned} \nabla\eta_1 &= \gamma\nabla\eta , \\ \eta_1 &= \gamma\eta + h_0 , \\ h &= (1 + \gamma)\eta + h_0 . \end{aligned} \tag{17}$$

The arbitrary constant h_0 is determined from the specified volume V_0 of the upper layer, or from an equivalent condition.

The equations for the two-layer ocean, analogous to equations (5), (6), and (9) for the homogeneous ocean, are

$$\mathbf{v}h = \nabla \times (\psi\mathbf{k}) , \tag{18}$$

$$Q = Q(\psi) = g(h - h_0)/(1 + \gamma) + \tfrac{1}{2}(\mathbf{v}\cdot\mathbf{v}) , \tag{19}$$

$$\zeta_a'/h = - dQ/d\psi . \tag{20}$$

The corresponding nondimensional equations for the two-layer ocean are formed by introducing the additional nondimensional variables h', ψ', σ', defined as $h' = h/h_0$, $\psi' = \psi/Uh_0L_0$, $\sigma' = \sigma/(U/L_0)$, and the nondimesnional parameter α, defined as $\alpha = (1 + \gamma)\eta_0/h_0$.

The nondimensional equations which form the dynamic constraints for free motion in the two-layer ocean are

$$\zeta_a'/h' = - dQ'/d\psi' , \tag{21}$$

$$Q'(\psi') = (1/\alpha)(h' - 1) + \tfrac{1}{2}\delta(\mathbf{v}'\cdot\mathbf{v}') , \tag{22}$$

where $\zeta_a' = f' + \delta\zeta'$ and $h' = 1 + \alpha\eta'$.

Equations (21) and (22) contain three unknown functions each and must be solved simultaneously. Thus, in the two-layer ocean, (22) is restrictive in contrast with (11) which is not restrictive. As in the homogeneous ocean, the circulation is not uniquely determined, because ζ_a'/h' can be any function of ψ'.

No solutions of the equations for the two-layer ocean have been obtained. The circulation in the upper layer will resemble the circulation in the homogeneous ocean if the divergence σ' can be neglected.

The divergence is small if the change in thickness of the upper layer along a streamline is small compared with the total thickness of the upper layer, i.e., if α is small compared with unity.

In high-velocity regions of the two-layer ocean the thickness of the upper layer is reduced, so that the absolute vorticity is smaller in magnitude than the absolute vorticity in a corresponding region of the homogeneous ocean. For a two-layer ocean located in the northern hemisphere, the relative vorticity must be less than the relative vorticity in the homogeneous ocean for corresponding flows. Therefore, the intensification of a current flowing toward the pole is more pronounced and that of a current flowing toward the equator is less pronounced than the intensification of a corresponding current in the homogeneous ocean.

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SALT BUDGET IN THE LAKE WASHINGTON SHIP CANAL SYSTEM[1]

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ABSTRACT

Salt water enters the freshwater system of Lake Washington, Montlake Canal, Lake Union, Fremont Canal, and Salmon Bay through the U. S. Government Locks at Ballard. During the summer season of heavy lock operation and small runoff this salt water accumulates in Lake Union and Lake Washington. Lake Union is flushed corresponding to the rates of flow during periods of high runoff, and Lake Washington is flushed only during the period of winter overturn. With increased chlorinity in the bottom waters, overturn and therefore flushing may be prevented. At present approximately 25% of the salt in Lake Washington is flushed out annually.

INTRODUCTION

The Lake Washington Ship Canal extends from Puget Sound to Lake Washington via the U. S. Government Locks at Ballard, Salmon Bay, Fremont Canal, Lake Union, and Montlake Canal (see Fig. 1). Lake Washington is nominally a freshwater lake while Puget Sound contains slightly diluted ocean water. The navigation channel is dredged and maintained everywhere at a minimum depth of 9 m. Salmon Bay contains a catch basin 600 m long, 75 m wide, and 15 m deep from which a siphon 2.7 m^2 in cross section extends to the downstream side of the spillway, where water is discharged from the basin at an estimated 2.8 to 4.2 m^3/sec. Lake Union has an area of 3.7 km^2 with depths of 14 to 15 m in north and south basins and approximately 12 m over the separating sill. Lake Washington is 29 km long with an approximate area of 90 km^2 and a maximum depth of 65 m. The main tributaries are the Cedar and Sammamish Rivers which enter respectively at the south and north end of Lake Washington. The minimum discharge of each river, approximately the same, is less than 3 m^3/sec and occurs in either August or September. The maximum

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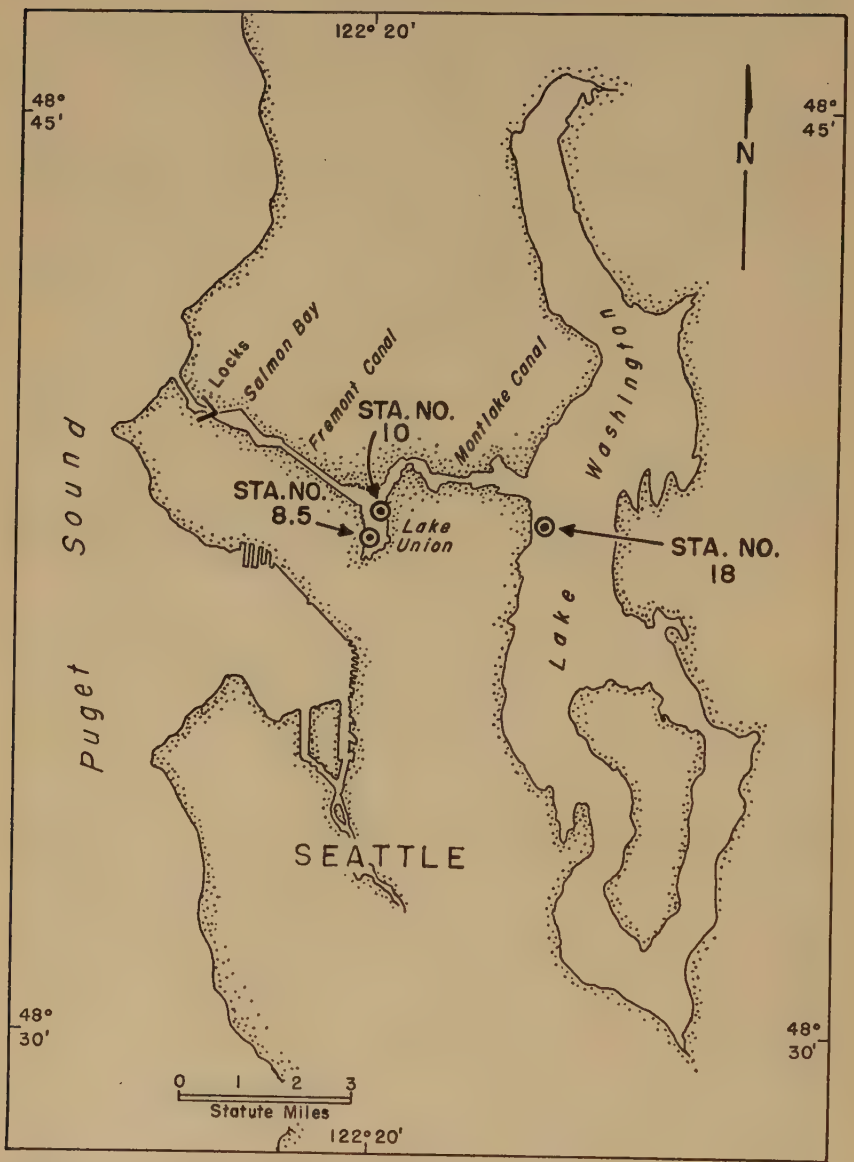

Figure 1. Location map showing hydrographic stations in Lake Union and Lake Washington.

discharge of the Cedar River is between 55 and 140 m^3/sec while that of the Sammamish River is 20 to 50 m^3/sec; the maxima occur in February or March.

SALT INTRUSION

The mechanism of salt intrusion into the Lake Washington Ship Canal system by lockage operations has been discussed previously by Smith and Thompson (1927), who showed that the water in the locks, just prior to raising the level, is always sea water. In raising the water in the locks from sea to lake level, lake water, introduced into the locks through culverts at the bottom of the chamber, actually "bubbles" through the sea water. Although mixing is not complete, the water introduced from Salmon Bay becomes more saline and the sea water becomes diluted. After the lock level is raised to that of the lake, the upper gates are opened to the canal side. The impounded water, being denser than that at the same level on the lake side, flows rapidly into Lake Union and is replaced by lighter surface water. The volume of Salmon Bay water used in each lockage is approximately twice that required to raise the lock level. During periods of light runoff and heavy lockages, the salt water in Lake Union may rise above sill level and spill over into Lake Washington.

To investigate the salt budget of this system, water samples were taken every two or three weeks in Lake Washington (depth 63 m) and in the two deep parts of Lake Union (depths 15 m). Sampling began in October 1950.

LAKE UNION

To illustrate the annual salt intrusion and flushing, the isochlors of 0.5, 1, 2, 5, and 7 ‰ at the northern Lake Union station are plotted in Figs. 2*a*, 2*b* and 2*c* from January 1951 through September 1953; shown also is the total rate of discharge through the canal system, averaged over a five-day period. Since computations on the exact amount of water used in lockage operations for 1953 were not available, the average monthly amounts for lockages in 1952 were used for 1953. The error is not significant during periods of large runoff, but a dashed line indicates the approximate nature of the curve after July. The dashed line in the chlorinity profiles indicates the approximate depth at and below which dissolved oxygen was absent and hydrogen sulfide generally present.

Starting with 10 January 1951, the 0.5 ‰ isochlor was at 12.5 m; subsequently it rose to 11 m and then disappeared completely during mid-February. During this period the isochlors were rather closely spaced, and the stagnant water which was present in the lower layers also disappeared in February.

The curve for total flow, which also showed some variation during January, reached a peak during February, when a flow of more than 100 m³/sec lasted for 13 days. The highest 5-day average flow during this time was 290 m³/sec, with the peak flood of 300 m³/sec on 13 February.

Soon after the lake was flushed, salt water began to reappear more or less in steps. These steps of sharp increases in chlorinity seemed to coincide with decrease in flow, which remained generally above 49 m³/sec for the first 25 days of March. After March, as the runoff

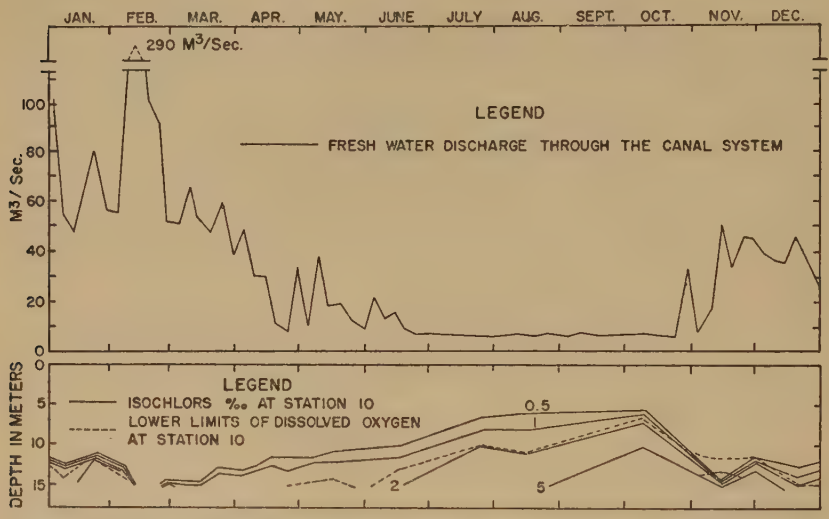

Figure 2a. Chlorinity (‰) variation at station 10 in Lake Union, and freshwater discharge (m³/sec) through the Canal, 1951.

through the system decreased, the amount of salt increased rapidly, particularly in concentrations between 1 and 2 ‰. Dissolved oxygen also decreased rapidly, and at the end of April hydrogen sulfide began to reappear.

After 16 June no more water was wasted either over the spillway or through the culverts, and the intrusion of salt water was indicated by a steady increase of chlorinity in the lake throughout the summer and early autumn. At the same time the amount of stagnant water increased at approximately the same rate.

With the return of the rainy season in early October, water is again wasted at the locks, depending on the lake level. In 1951, as soon as the flow had increased to approximately 33 m³/sec (14 November), the

5 ‰ isochlor dropped sharply to 14 m; by 29 November it was back at 11 m.

This general pattern of flushing during the winter and of increasing chlorinity during the summer was repeated in 1952 and 1953 except for the complete flushing that took place in February 1951. Precipitation during the winter and spring of 1952 was below average, and as can be seen in Fig. 2, wastage of water was well distributed throughout the first six months of the year, the amounts being considerably below 50 m³/sec most of the time. Only once, for a brief period of 16 days

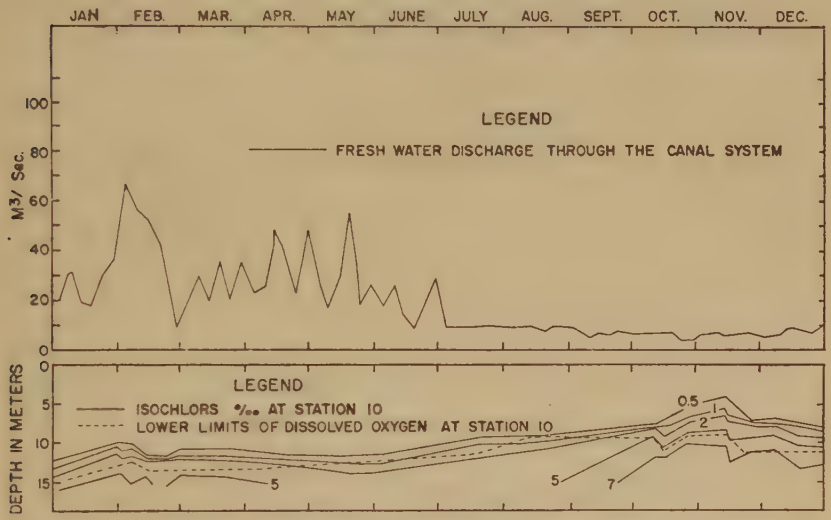

Figure 2*b*. Chlorinity (‰) variation at Station 10 in Lake Union, and freshwater discharge (m³/sec) through the Canal, 1952.

in February, did discharge exceed this. Though the isochlors dipped for this period, they rose again shortly thereafter during a rather low runoff period. During April and May there were three brief periods of flow greater than 40 m³/sec which were reflected in a lowering of the interface by approximately 1 m.

After mid-May 1952 the isochlors began to rise steadily. From July through the rest of the year the water flowing through the system was only that used in lockage operations, fish ladder, and siphon. As in 1951, the 5 ‰ isochlor appeared at the end of August. Subsequently, however, the chlorinity increased rapidly and the 7 ‰ isochlor appeared below 10*m*. The complete closure of the saltwater siphon from 5 to 19 September and its partial closure from 19 September to 5

December probably account for the appearance of this abnormally high chlorinity.

Noteworthy is the sudden drop of the 0.5 ‰ isochlor after 12 November 1952 and its continuation downward throughout December, although no wastage of water occurred until 7 January 1953. Notwithstanding a flow greater than 80 m³/sec for 30 days, the interface level did not drop below 10 m until March. The 7 ‰ isochlor level dropped slowly during the first three months of 1953 until May, when it disappeared. The 0.5, 1, and 2 ‰ isochlors began to rise during May, as in previous years, while the 5 ‰ isochlor continued to decline.

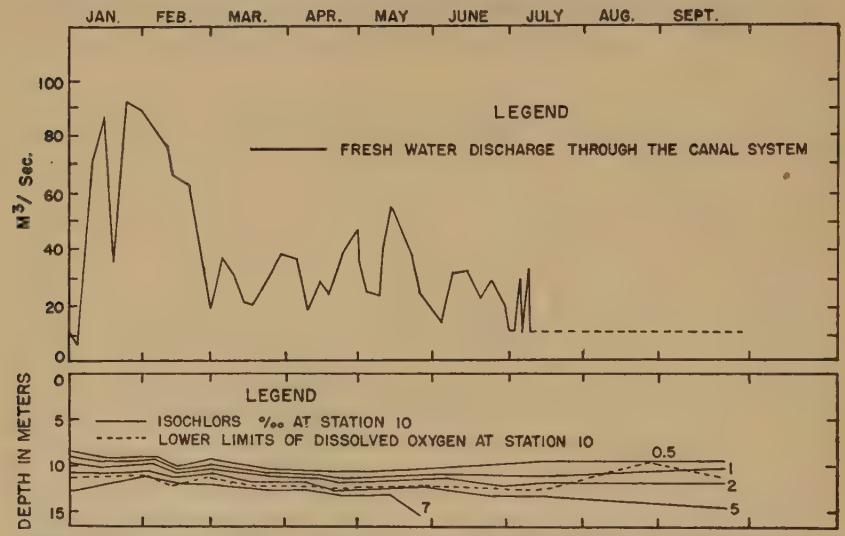

Figure 2c. Chlorinity (‰) variation at Station 10 in Lake Union, and freshwater discharge (m³/sec) through the Canal, 1953.

It is concluded that Lake Union will be flushed completely when the discharge through the system is in excess of 100 m³/sec for several days, as was the case in February 1951. Partial flushing, which takes place when the discharge is greater than 50 m³/sec, seems to depend not only on the density difference between the upper and lower zones but also on the density gradient below the interface. For example, since the density gradient in October 1951 was considerably smaller than it was during the following winter and spring, a rate of flow of 50 m³/sec produced a greater dip in the interface than did 65 m³/sec the following February. It is assumed that the water above sill level (10 m) drained out when the flow increased, while flushing by the process of mixing extended to 4 m below that depth.

The importance of density difference can be seen in a comparison of observations for February 1952 with those for February 1953 (Fig. 2). In each case the interface was at approximately the same depth, but the chlorinity in the saltwater layer in 1953 was higher than that in 1952. A flow of more than 82 m^3/sec in 1953 did not flush the lake any more than the 65 m^3/sec in the previous year.

Apparently a flow of 30 to 50 m^3/sec does some flushing over a period of two to three months, as was the case in April and May of 1952 and 1953, when the interface was lowered by approximately 1 m. However, this rate of flow does not prevent new salt water from flowing into the lake, as was the case during the spring of 1951 and apparently during December 1951 and January 1952.

Three phenomena require special explanation. First, the rapid reappearance of salt water after the initial flushing in November, as indicated by lack of oxygen, is due to drainage from the southern part of the lake rather than to inflow from the lock system. Second, the decrease in the level of the interface commencing in November 1952 could not be caused by mixing, considering the small flow during this period. The maintenance of salt water above sill level must result from a balance between the inflow from the locks and outflow into both Salmon Bay and Lake Washington, due to the salt water head. Thus the decrease in lockage volume, especially on the 13th, 16th and 20th, explains the observed drop in the interface level. Third, the disappearance of the 7 ‰ isochlor in May 1953 while the interface level was rising indicates internal mixing within the saltwater layers while there is little or no mixing across the interface. This mixing probably results from the initial energy of the incoming salt water.

LAKE WASHINGTON

The limnology of Lake Washington has been discussed by Sheffer and Robinson (1939). Lake Washington, a typical freshwater lake, has minimum temperatures of approximately 6° to 8° C, depending on the climatic conditions of the year. Because of these high minimum temperatures, the lake overturns during one extended period each year, beginning approximately in December and lasting through February or March. After March the lake rapidly becomes stratified and maximum surface temperatures of 20° to 25° C occur during August.

Chlorinities reported in early studies of this lake were very low. The chlorinity reported by Sheffer and Robinson was approximately 2 p.p.m. in 1933 while that listed by Smith and Thompson (1927) did not exceed 4 p.p.m. The latter authors also listed the total salt analysis made by J. G. Priestley, chemist for the city of Seattle, from

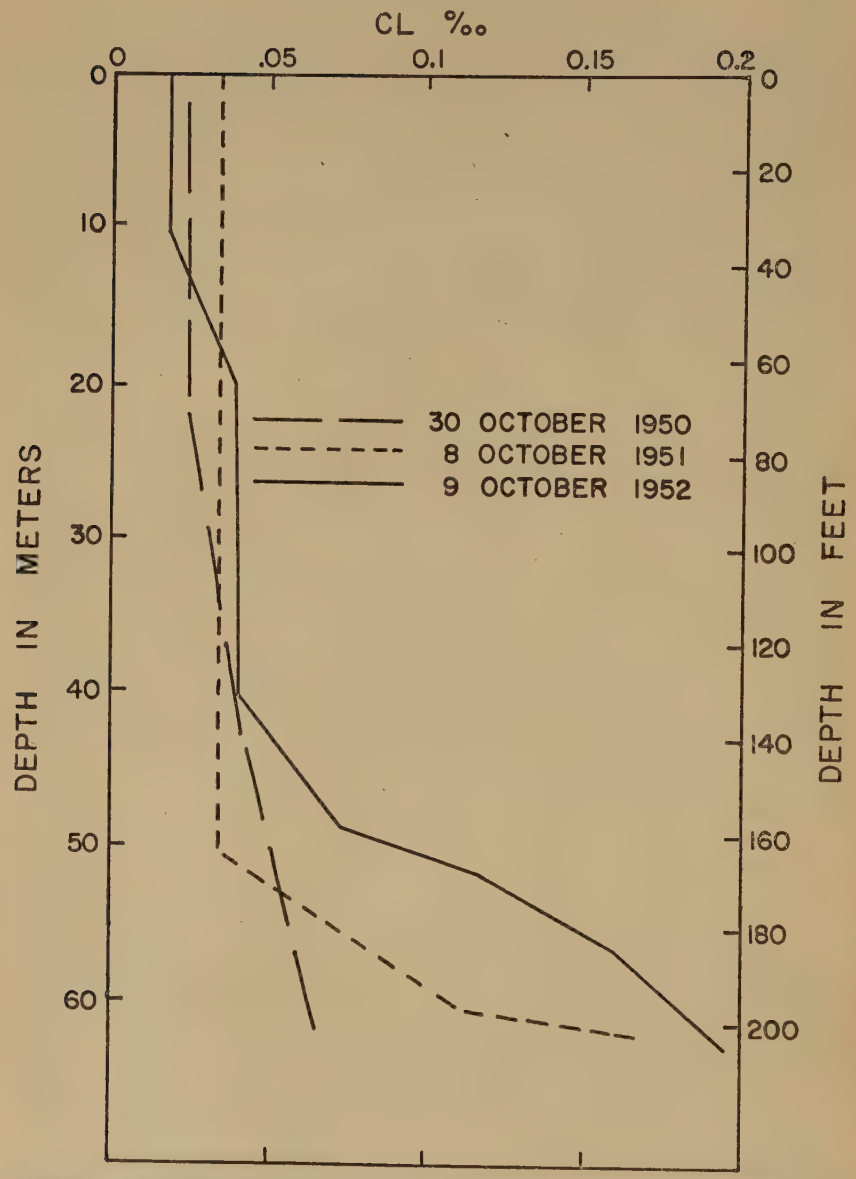

Figure 3. Chlorinities (‰) in Lake Washington (Station 18) for 30 October 1950, 8 October 1951, 9 October 1952.

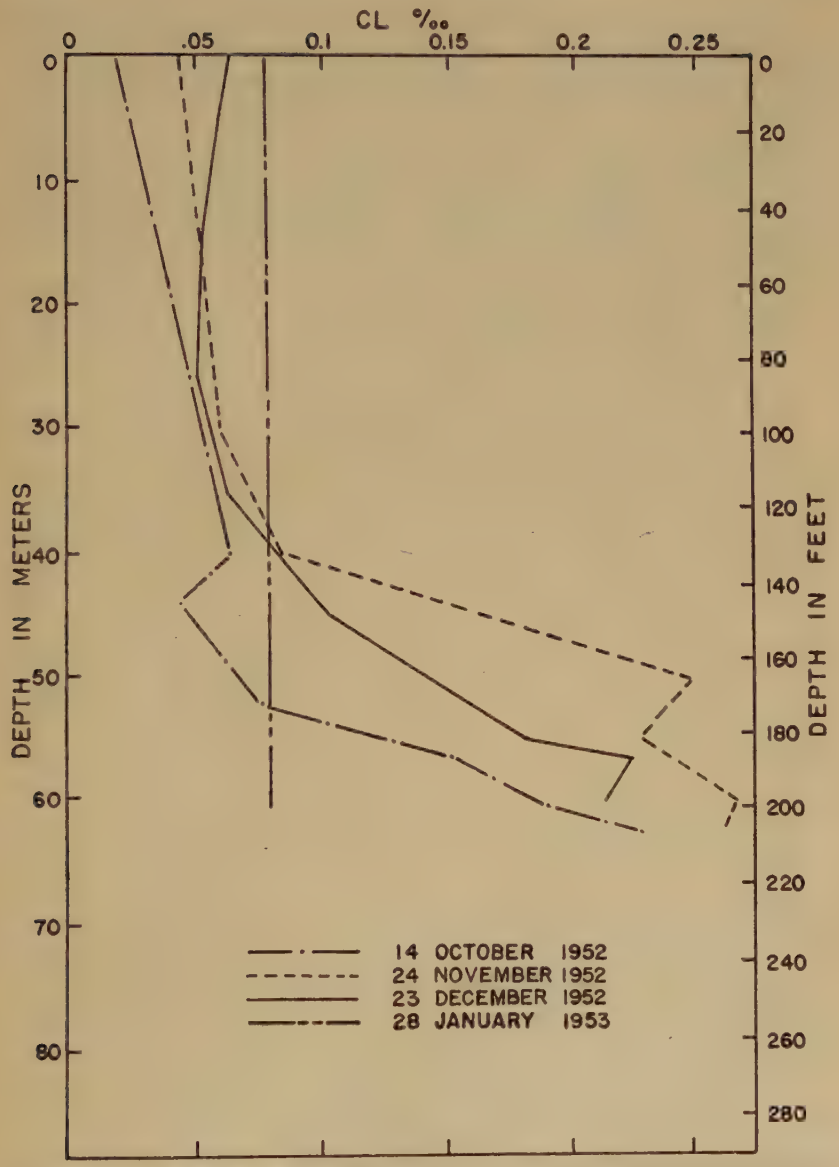

Figure 4. Chlorinities (‰) in Lake Washington (Station 18) for 14 October, 24 November 23 December 1952, 28 January 1953.

whose data the sulfate-chlorinity ratios were computed to be 4.2 and 6 for the north and south ends of the lake respectively. These ratios are typical for fresh water, and it is safe to say that at least until 1933 there was no appreciable saltwater intrusion into Lake Washington.

During the past few years the minimum chlorinity of the lake has been at least five times that found by Smith and Thompson. In Fig. 3 the chlorinity profiles for October 1950, 1951 and 1952 indicate this growth in total salt content, both the maximum chlorinity at the

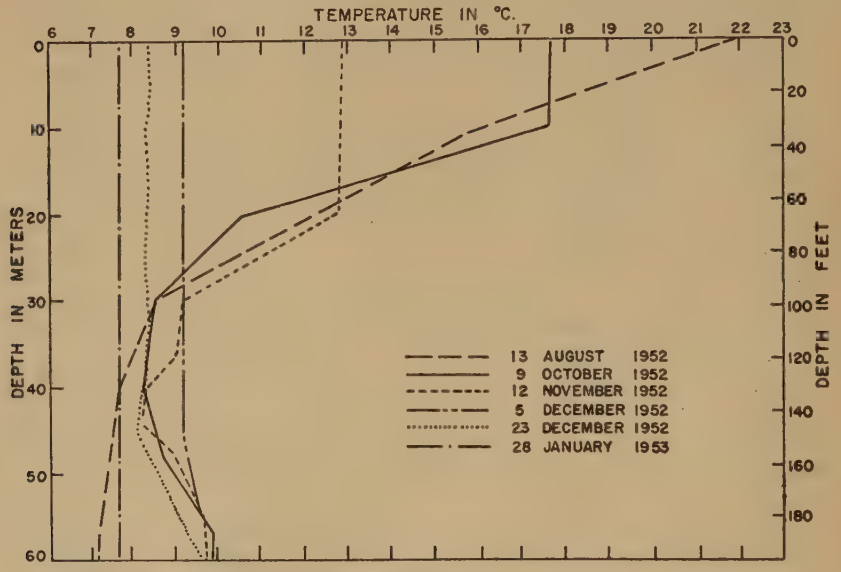

Figure 5. Temperatures (° C) in Lake Washington (Station 18), Autumn 1952.

bottom and the height of the brackish water layer having increased each succeeding year. This increase occurs when the interface in Lake Union is above sill level and the rate of discharge of fresh water is too low to prevent the flow of saline water eastward along the bottom of Montlake Canal. At higher freshwater discharge rates, mixing and shearing stresses across the interface prevent salt water from reaching Lake Washington. Stagnation, which will occur when the bottom layers become sufficiently dense to prevent overturn during the winter season, is thus likely in following years.

The minimum requirements for prevention of overturn are at present unknown, but some light can be thrown on them by examination of the temperature and salinity profiles in Lake Washington during the

autumn of 1952 and winter of 1953 (Figs. 4 and 5). With the bottom water at 10° C and about 0.25 ‰ chlorinity and with the surface water at 8.4° C and 0.05 ‰, no overturn occurred; however, when the surface temperatures reached 7.7° C on 28 January, an overturn did occur. Since 7.7° C is within a few degrees of the minimum lake temperature and since the density of the water varies slowly with temperature near that of maximum density, the chlorinity of the bottom water must be close to that which is sufficient to prevent overturn. Knudsen's Tables (1901), while not suitable for determining actual density values for low chlorinities, will give satisfactory results for the density difference between surface and bottom waters; therefore they can be used to estimate the maximum allowable chlorinity of the bottom waters. From the above chlorinity and temperature values, we find that overturn occurred between top to bottom density differences of 1.3×10^{-4} and 0.7×10^{-4} g cm^{-3}. If we assume that overturn occurs at a density difference of 1.0×10^{-4} g cm^{-3}, and if we consider a minimum surface temperature of 6° C while that at the bottom remains at 10° C, then the maximum bottom chlorinity for overturn would be 0.28 ‰. We can set a definite upper limit for the density difference at 1.3×10^{-4} g cm^{-3}, which then allows a bottom chlorinity of 0.30 ‰. Of course, at lower values of bottom chlorinity, a mild or less stormy winter might fail to cause overturn. The problem of salt water intrusion into Lake Washington is thus critical with regard to the annual flushing process of the lake.

Since the total chloride content of the lake showed an increase each year during this survey, it is possible that problems of salt water contamination might arise regardless of overturn. To investigate this phase of the problem, the annual salt budget for the lake is given. Since the salt content of the entire lake is determined from measurements at only one station, some error is inevitable. However, there are two times of the year when observation should give a fairly high degree of accuracy: first, in December or January when the water has become homogeneous due to overturn and wind mixing and before much water has been discharged; second, in July after wastage of water has ceased and thermal stratification has set in. Since the major flushing action occurs between December and July when there is no salt inflow, a comparison of total chloride content should give a reasonable approximation for the total flushing of the lake; Table I gives the total chloride content and the percent flushing from winter to summer for the years 1951–1953. The flushing, about 20% for the winter-to-summer period each year, is apparently unrelated to the total salt content. An independent check on this value can be made if it is assumed that the salt removal is accomplished by the outflow of surface

TABLE I. Lake Washington Total Chloride Content and Percent Flushing

	Cl *1000 tons*	*winter-summer*
27 December 1950	88	17
24 July 1951	73	
19 December 1951	102	20
18 July 1952	81	
28 January 1953	230	20
15 July 1953	183	

TABLE II. Lake Washington Flushing Based on Surface Outflow

1953	10^6 m^3	*1000 tons Cl*
January	158	12.8
February	167	13.1
March	76	4.0
April	79	4.5
May	82	4.9
June	62	3.7
July	31	1.8

water through the Montlake Canal. Table II gives the monthly outflow and flushing for the first seven months of 1953, the figures being based on the mean monthly chlorinity of the upper 15 m. These values are also plotted as a dashed line in Fig. 6 for 7 January, when it is assumed that salt intrusion ceased, through 28 January 1953 (230,000 tons). The curve based on data from station 18 approaches the computed curve in July and finally meets it in November after salt intrusion has taken place. Since the two curves join in January and then approach each other closely in July, the salt flushing can be explained by the surface runoff, hence the flushing over a twelve-month period can be determined on this basis. Thus the total chloride flushed out of the system (approximately 57,000 tons) is computed to be 24% of the maximum chloride content (240,000 tons) at the beginning of the year. This is a reasonable figure when it is considered that the annual flow through the lock system is approximately 30% of the lake volume.

An expression for the maximum salt content of the lake during any one year can be written in terms of the salt content during some initial year and the inflow for each intervening year. If we use a figure of 25% for the annual flushings, we obtain

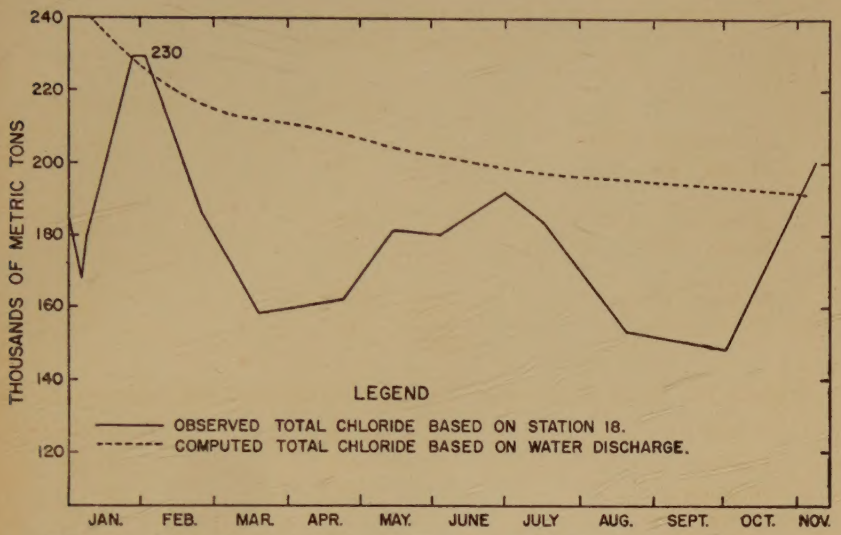

Figure 6. Total chloride (metric tons) in Lake Washington (Station 18), 1953

$$C_i = (0.75)^i C_0 + \sum_{j=0}^{i-1} (0.75)^j B_{i-j},$$

where C_i is the maximum content at the end of the ith year, C_0 is the initial content, and B_j is the inflow during the jth year.

This relation can be used to illustrate the extent of the present rate of salt inflow into the lake. For example, if we consider a 150,000-ton annual rate of inflow, as in 1952, then the equilibrium content will be 600,000 tons or more than 2½ times that which actually occurred in 1952. Or, if a maximum content of 230,000 tons is used, then the annual inflow will be restricted to 57,500 tons. Now, if the content were to reach 400,000 tons and if the annual inflow were to drop to 50,000 tons per year, the content after four years would be decreased only to 263,000 tons. Thus it is evident that one bad year requires four or five years of less inflow to bring the content back down to its original level.

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